IMA Journal of Mathematics Applied in Business & Industry (1992) 3, 377–386

The maintenance of bidirectionally patrolled machines

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[Received 30 April 1991 and in revised form 16 January 1992]

The problem considered is that of maintaining a set of N stations (machines, production facilities) which are set out along a line and numbered, say, from left to right, 1 to N. The stations (which are not necessarily identical) are maintained and repaired if necessary by one operative, who patrols them first from left to right in the order 1 to N and then from right to left in the order $N - 1$ down to 1 and so on.

It is assumed that breakdowns at station i occur completely at random in running time at an average rate λ_i. The time for the operative to travel from left to right from station $i - 1$ to station i (or from right to left from station i to station $i - 1$) and then to carry out routine maintenance at station i is assumed to be a constant for this pair of stations, and is denoted by w_i. If, on arrival at station i, the operative finds the station out of action, then an additional time r_i is needed to repair station i. It is assumed that r_i is a constant for station i. It is also assumed that a repair attempt at station i is successful with probability σ_i (not necessarily 1). Thus the model caters for a heterogeneous set of stations, unequally spaced.

Important performance measures for the system include the average time to traverse the line of stations, along with the mean availability. For individual stations, the availability, the mean time spent waiting for attention, and the mean length of the stopped period are all important. It is shown how all of these quantities can be computed.

1. Introduction

We consider the problem of the maintenance of a set of N stations, arranged in a line and numbered, say from left to right, 1 to N. The stations are maintained by one operative who patrols them first from left to right in the order 1 to N, and then from right to left in the order $N - 1$ down to 1, and so on, as in the following diagram.

x————x———x— - - - - - - - - - - - - ———x————x————x

1 2 3 $N-2$ $N-1$ N

In many factory-floor situations, this problem arises quite literally with the machines arranged in rows. However, the model could equally describe the situation where the stations are facilities located in different cities but needing regular maintenance as well as occasional repair. The operative visits each station in turn, and then returns to his starting point in reverse order for reasons of travelling convenience.

It is assumed that breakdowns at station i occur completely at random, i.e. in a Poisson process, in running time at average rate λ_i. The time for the operative to travel from left to right from station $i-1$ to station i (or in the reverse direction) and then to carry out routine maintenance at station i (or $i-1$), referred to as the *travel* time, is assumed to be a constant for each station pair $(i-1, i)$ and is denoted by w_i. If, on arrival at station i, it is found to be broken down, then an additional time r_i is required to repair it and return it to the running state. It is assumed that r_i is a constant for station i.

The problem is thus a particular case of the machine interference problem. If the travel time were neglected, it could be treated by the methods first used by Ashcroft (1950) who gave a solution to the M/G/1/N machine interference problem. Attempts to take account of the travel time, and various patrolling disciplines in the homogeneous case, include the work of Mack, Murphy & Webb (1957), Bunday & Mack (1973), and Bunday & El-Badri (1984).

More recently, considerable interest in this type of problem has arisen in the field of computer performance modelling and evaluation. Here messages arrive at station i in Poisson fashion at an average rate λ_i. Messages which arrive at a blocked station are lost. The time to transmit a message from station i is r_i, and the switch-over time between adjacent stations is w_i. A good review and bibliography of some of these applications is given by Takagi (1986, 1988, 1990). The particular back-and-forth patrolling discipline considered in this paper has been used to model the reading/writing of information from/to a disk. The operative constitutes the read/write head which moves back and forth along a radius of the disk; the stations correspond to bands on the disk. Models concerned with this so-called SCAN system include the work of Coffman & Hofri (1982), Swartz (1982), and Coffman & Gilbert (1987) who investigated a continuous model.

We allow for the possibility of unsuccessful repairs. It is assumed that a repair attempt at station i is successful with probability σ_i. This aspect of the model arises quite naturally in some industrial applications. The operative is a robot patrolling the line of stations (machines) and sometimes fails in its repair attempt. In that situation, that station remains out of action at least until the next visit and repair attempt. Yet another interpretation is that the repair consists of a number of phases and that, at each visit, one phase is successfully carried out. The same mathematical model arises if the number of phases of repair at station i has a geometric distribution with mean $1/\sigma_i$. Thus the completion of a phase corresponds to the completion of the total repair with probability σ_i.

2. The mathematical model

Since breakdowns at station i occur completely at random at average rate λ_i, residual run times at station i have an exponential distribution with mean $1/\lambda_i$. Thus if station

i is left running at time t, the probability that it is still running after a further time T is

$$\exp(-\lambda_i T), \tag{2.1}$$

whereas the probability that it is then stopped is

$$1 - \exp(-\lambda_i T) \tag{2.2}$$

independent of t. For $u_i \in \{0, 1\}$ $(i = 1, ..., N)$, let $\boldsymbol{u}$ denote the vector $(u_1, ..., u_N)$, and denote by $\vec{\boldsymbol{u}}$ the left-to-right traverse of the stations (i.e. in the order 1 to N) in which the operative *leaves* station 1 in state u_1 and *finds* station i $(i = 2, ..., N)$ in state u_i, where $u_i = 0$ denotes that station i is running and $u_i = 1$ denotes that station i is broken down. We write $\pi(\vec{\boldsymbol{u}})$ for the probability that the operative encounters this state on a left-to-right traverse in the steady-state situation. Similarly, $\overleftarrow{\boldsymbol{v}}$ denotes the right-to-left traverse, in the order N down to 1, in which the operative *leaves* station N in state v_N, and *finds* station $N-k$ $(k = 1, ..., N-1)$ in the state v_{N-k}. The probability that the operative encounters these states on a right-to-left traverse in a steady-state situation is correspondingly written $\pi(\overleftarrow{\boldsymbol{v}})$. The 2^{N+1} probabilities $\pi(\vec{\boldsymbol{u}})$ and $\pi(\overleftarrow{\boldsymbol{v}})$ $(\boldsymbol{u}, \boldsymbol{v} \in \{0, 1\}^N)$ satisfy the normalization conditions

$$\sum_{\boldsymbol{u}} \pi(\vec{\boldsymbol{u}}) = 1, \qquad \sum_{\boldsymbol{v}} \pi(\overleftarrow{\boldsymbol{v}}) = 1. \tag{2.3}$$

In addition,

$$\pi(\vec{\boldsymbol{u}}) = \sum_{\overleftarrow{\boldsymbol{v}}} \pi(\overleftarrow{\boldsymbol{v}}) p(\overleftarrow{\boldsymbol{v}}, \vec{\boldsymbol{u}}), \qquad \pi(\overleftarrow{\boldsymbol{v}}) = \sum_{\boldsymbol{u}} \pi(\vec{\boldsymbol{u}}) p(\vec{\boldsymbol{u}}, \overleftarrow{\boldsymbol{v}}), \tag{2.4a,b}$$

where, in (2.4a), the transition probability $p(\overleftarrow{\boldsymbol{v}}, \vec{\boldsymbol{u}})$ is the conditional probability, given the right-to-left traverse $\overleftarrow{\boldsymbol{v}}$, that the left-to-right traverse $\vec{\boldsymbol{u}}$ follows immediately, while (2.4b) has a similar interpretation since right-to-left traverses will follow left-to-right traverses.

The transition probability in (2.4a) can be calculated by considering in turn the transitions at the individual stations. For station j, the time that elapses between the operative leaving it on a right-to-left traverse and next visiting it on the following left-to-right traverse is

$$\tau_j = \begin{cases} 2w_2 + r_1 v_1 & \text{for } j = 2, \\ 2\sum_{i=2}^{j} w_i + r_1 v_1 + \sum_{i=2}^{j-1} r_i(u_i + v_i) & \text{for } j = 3, ..., N. \end{cases}$$

Given that station j was found in state v_j with station $j-k$ in state v_{j-k} $(k = 1, ..., j-1)$ on a right-to-left traverse, and that station i was found in state u_i $(i = 1, ..., j-1)$ on the successive left-to-right traverse, the conditional probability that station j will be found in state u_j on the latter traverse is denoted by $q_j(v_j, u_j)$, for $j = 2, ..., N$. Here $q_j(v_j, u_j)$ depends on $u_1, ..., u_{j-1}$ and $v_1, ..., v_{j-1}$ through the quantity τ_j, but this dependency is suppressed in notation for simplicity. If we use (2.1) and (2.2), we obtain

$$\begin{aligned} q_j(0,0) &= \exp(-\lambda_j\tau_j), & q_j(0,1) &= 1-\exp(-\lambda_j\tau_j), \\ q_j(1,0) &= \sigma_j\exp(-\lambda_j\tau_j), & q_j(1,1) &= 1-\sigma_j\exp(-\lambda_j\tau_j), \end{aligned} \tag{2.5}$$

for $j = 2,\dots, N$. For $j = 1$, we need the probability of *leaving* station 1 in state u_1 having just *found* it in state v_1. Thus

$$q_1(0,0) = 1, \quad q_1(0,1) = 0, \quad q_1(1,0) = \sigma_1, \quad q_1(1,1) = 1-\sigma_1. \tag{2.6}$$

For $j = N$, we need the probability of *finding* station N in state u_N, having last *left* it τ_N time units earlier in state v_N. Thus

$$\begin{aligned} q_N(0,0) &= \exp(-\lambda_N\tau_N), & q_N(0,1) &= 1-\exp(-\lambda_N\tau_N), \\ q_N(1,0) &= 0, & q_N(1,1) &= 1. \end{aligned} \tag{2.7}$$

We use (2.5)–(2.7) to compute the transition probabilities in (2.4a) as

$$p(\vec{v}, \vec{u}) = \prod_{j=1}^{N} q_j(v_j, u_j).$$

A similar analysis allows the computation of the transition probabilities in (2.4b). Then (2.3)–(2.4) supply us with 2^{N+1} independent equations for the 2^{N+1} state equilibrium probabilities $\pi(\vec{u})$ and $\pi(\vec{v})$. At the moment, no easily computed closed-form solution for these equations appears to exist. They have defied our own ingenuity as well as that of several computer algebra packages. However, numerical solutions to any specified accuracy can be obtained, although even here we were restricted to values of $N \leqslant 9$ because of the limitations of computing facilities available to us.

3. Some quantities of interest

The average time τ taken by the operative to traverse the stations in a left-to-right traverse is given by

$$\tau = \sum_{i=2}^{N} w_i + \sum_{\vec{u}} \left(\pi(\vec{u}) \sum_{j=2}^{N} u_j r_j \right). \tag{3.1}$$

In precisely the same way we can find the mean time τ' for a right-to-left traverse.

In the equilibrium state, the probability that station j is found in the running state when approached from the left is denoted by

$$z_j = \sum_{\vec{u}: u_j = 0} \pi(\vec{u}) \quad (j = 2,\dots, N), \tag{3.2}$$

where the summation is over those states for which $u_j = 0$. The probability that station j is found running when approached from the right is denoted by z'_j, where

$$z'_j = \sum_{\vec{v}: v_j = 0} \pi(\vec{v}) \quad (j = 1,\dots, N-1). \tag{3.3}$$

For $j \geqslant 2$, a *left partial cycle* (LPC) of station j means a journey that the operative makes between departure (leftwards) from station j and the next return to it via station 1. A *left partial cycle time* (LPCT) of station j is the time taken for such a journey; note that this does not include any time spent at station j. For $j \leqslant N-1$, a *right partial cycle* (RPC) and *right partial cycle time* (RPCT) are defined similarly when the journey, initially rightwards, is via station N. Let $a(\mathcal{L}, j)$ $(j = 2, \ldots, N)$ denote the probability that, in an LPC of station j, the set $\mathcal{L}$ of stations is found stopped where

$$\mathcal{L} \subseteq \mathcal{L}_j = \{1', \ldots, (j-1)'\} \cup \{2, \ldots, j-1\},$$

the primed integers referring to the stations visited on the first (right-to-left) pass, and the unprimed integers to those visited on the second (left-to-right) pass. Corresponding to the event $\mathcal{L}$, the LPCT of station j takes the value

$$t(\mathcal{L}, j) = 2\sum_{i=2}^{j} w_i + \sum_{i \in \mathcal{L}} r_i, \tag{3.4}$$

where $r_{1'}$ is identified with r_1, etc. Similarly $a_r(\mathcal{L}, j)$ denotes the conditional probability that an LPC of station j encounters the state $\mathcal{L}$, given that station j was *running* when approached from the right just prior to the left partial cycle, whereas $a_s(\mathcal{L}, j)$ denotes the conditional probability that an LPC encounters the state $\mathcal{L}$, given that station j was found *stopped* immediately prior to the left partial cycle.

For station 2, we have

$$a(\varnothing, 2) = z_1', \qquad a(\{1'\}, 2) = 1 - z_1', \tag{3.5}$$

and, for $j = 3, \ldots, N-1$,

$$a(\mathcal{L}, j) = z_j' a_r(\mathcal{L}, j) + (1 - z_j') a_s(\mathcal{L}, j). \tag{3.6}$$

Further we have

$$\begin{aligned} z_j = z_j' &\sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_r(\mathcal{L}, j) \exp[-\lambda_j t(\mathcal{L}, j)] \\ &+ (1 - z_j')\sigma_j \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_s(\mathcal{L}, j) \exp[-\lambda_j t(\mathcal{L}, j)] \quad (j = 2, \ldots, N-1), \end{aligned} \tag{3.7a}$$

$$z_N = z_N' \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a(\mathcal{L}, N) \exp[-\lambda_N t(\mathcal{L}, N)], \qquad z_1 = z_1' + \sigma_1(1 - z_1'), \tag{3.7b,c}$$

where z_N' is the probability that the operative leaves station N running on a right-to-left right traverse and z_1 is the probability that he leaves station 1 running on a left-to-right traverse. Of course the probabilities $a(\mathcal{L}, j)$, $a_r(\mathcal{L}, j)$, and $a_s(\mathcal{L}, j)$ form complete distributions, so that

$$\sum_{\mathcal{L} \subseteq \mathcal{L}_j} a(\mathcal{L}, j) = \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_r(\mathcal{L}, j) = \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_s(\mathcal{L}, j) = 1. \tag{3.8}$$

We can treat the RPCs in a completely analogous fashion and obtain a set of equations similar to (3.4)–(3.8) where, in the analogue of (3.7), the z_j' appear on the left and the z_j on the right.

For $j = 2, \ldots, N-1$, the mean time that station j actually runs during one of its LPCs is

$$L_j = z_j' \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_r(\mathcal{L}, j)\left(\int_0^{t(\mathcal{L},j)} \lambda_j x\, e^{-\lambda_j x}\, dx + t(\mathcal{L}, j) \exp[-\lambda_j t(\mathcal{L}, j)]\right)$$

$$+ \sigma_j(1 - z_j') \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_s(\mathcal{L}, j)\left(\int_0^{t(\mathcal{L},j)} \lambda_j x\, e^{-\lambda_j x}\, dx + t(\mathcal{L}, j) \exp[-\lambda_j t(\mathcal{L}, j)]\right).$$

Thus, on carrying out the integration and using (3.7a) and (3.8), we readily obtain

$$L_j = z_j' \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_r(\mathcal{L}, j) \frac{1 - \exp[-\lambda_j t(\mathcal{L}, j)]}{\lambda_j}$$

$$+ \sigma_j(1 - z_j') \sum_{\mathcal{L} \subseteq \mathcal{L}_j} a_s(\mathcal{L}, j) \frac{1 - \exp[-\lambda_j t(\mathcal{L}, j)]}{\lambda_j}$$

$$= \frac{z_j' + \sigma_j(1 - z_j') - z_j}{\lambda_j}.$$

Similarly we can calculate R_j, the mean running time in an RPC for station j. Thus, if F_j denotes the mean running time of station j in a complete cycle, then

$$F_j = L_j + R_j = \frac{\sigma_j(1 - z_j + 1 - z_j')}{\lambda_j},$$

for $j = 2, \ldots, N-1$. Similar manipulation shows that

$$F_1 = \frac{z_1 - z_1'}{\lambda_1} = \frac{\sigma_1(1 - z_1')}{\lambda_1}, \qquad F_N = \frac{z_N' - z_N}{\lambda_N} = \frac{\sigma_N(1 - z_N)}{\lambda_N}.$$

4. Performance measures

If we use (3.2) in the expression (3.1) for the mean left-to-right traverse time, and also consider the analogous result for τ', then the mean time for a complete cycle (two successive traverses) of the stations can be written in the form

$$\tau + \tau' = 2 \sum_{i=2}^{N} w_i + \sum_{j=2}^{N} r_j(1 - z_j) + \sum_{i=1}^{N-1} r_i(1 - z_i').$$

Thus for station j, the availability A_j, defined as the proportion of time that the station is actually running, is given by

$$A_j = F_j/(\tau + \tau').$$

An important measure for station j is the mean time that it is out of action, i.e. the mean time from its breakdown to the moment its repair is completed. This we denote by $\mathrm{E}[T_j]$. We use $\mathrm{E}[W_j]$ to denote the mean waiting time of station j, i.e. the mean time that it stands idle awaiting the operative to work on its repair. These two quantities differ by the mean value of the repair time. Thus

$$\mathrm{E}[T_j] = \mathrm{E}[W_j] + r_j/\sigma_j.$$

But, since the mean run-time of station j is $1/\lambda_j$, we have the result

$$A_j = \frac{1}{\lambda_j}\Bigg/\left(\frac{1}{\lambda_j} + \mathrm{E}[T_j]\right),$$

whence

$$\lambda_j \mathrm{E}[T_j] = 1/A_j - 1.$$

Of course, even with homogeneous stations ($r_j = r$, $\lambda_j = \lambda$, $\sigma_j = \sigma$, $w_j = w$ at all stations), the performance measures above will still vary from station to station, depending on their position in the line. Those near the middle of the line have a higher availability than those positioned near the end of the line.

For the complete system the average availability, i.e. the proportion of the total possible time that stations are running, is given by

$$A = \frac{1}{N}\sum_{j=1}^{N} A_j.$$

5. The case of zero repair time

A simple exact analytical solution is possible in the case where $r_j = 0$ for all j. This will also serve as a good approximation to the situation where r_j is small compared to w_j, which could arise in the case where the stations are well spread out geographically. This model isolates the loss of availability due to repairs having to wait for the operative to reach the location. In this case, $t(\mathcal{L}, j)$ from (3.4) is simply $2\sum_{i=2}^{j} w_i$, so that, with

$$c_j = \exp[-\lambda_j t(\mathcal{L}, j)] = \exp\left(-2\lambda_j \sum_{i=2}^{j} w_i\right), \qquad c_j' = \exp\left(-2\lambda_j \sum_{i=j+1}^{N} w_i\right),$$

we find that the system (3.7) and its right-to-left analogue can be written as

$$z_j = z_j' c_j + (1 - z_j')\sigma_j c_j \quad \text{and} \quad z_j' = z_j c_j' + (1 - z_j)\sigma_j c_j' \quad (j = 2, \ldots, N-1),$$

$$z_N = z_N' c_N, \qquad z_N' = z_N + \sigma_N(1 - z_N),$$

$$z_1' = z_1 c_1', \qquad z_1 = z_1' + \sigma_1(1 - z_1').$$

It is clear that these equations can be treated in pairs as indicated and a closed-form exact solution obtained.

TABLE 1

Performance measures for selected values of the system parameters, with $N = 8$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.01	.90	10.00	.9672	.7003	42.80
2	1	.01	.90	10.00	.9236	.7522	32.94
3	1	.01	.90	10.00	.8792	.7718	29.57
4	1	.01	.90	10.00	.8369	.7818	27.91
5	1	.01	.90	10.00	.7968	.7818	27.91
6	1	.01	.90	10.00	.7592	.7718	29.57
7	1	.01	.90	10.00	.7239	.7522	32.94
8	1	.01	.90	10.00	.6719	.7003	42.80

$A = .7515$ $\tau = 21.09$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.01	.90	10.00	.9672	.7005	42.80
2	.5	.01	.90	10.00	.9323	.7482	33.65
3	2	.01	.90	10.00	.8713	.7741	29.18
4	.5	.01	.90	10.00	.8370	.7819	27.89
5	1	.01	.90	10.00	.7970	.7819	27.29
6	.5	.01	.90	10.00	.7663	.7741	29.18
7	2	.01	.90	10.00	.7174	.7482	33.65
8	.5	.01	.90	10.00	.6721	.7005	42.76

$A = .7512$ $\tau = 21.07$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.01	.90	10.00	.9671	.6993	43.00
2	2	.01	.90	10.00	.9064	.7587	31.80
3	.5	.01	.90	10.00	.8706	.7735	29.29
4	.5	.01	.90	10.00	.8362	.7813	27.99
5	1	.01	.90	10.00	.7961	.7813	27.99
6	.5	.01	.90	10.00	.7654	.7735	29.29
7	.5	.01	.90	10.00	.7367	.7587	31.80
8	2	.01	.90	10.00	.6700	.6993	43.00

$A = .7532$ $\tau = 21.18$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.01	.90	20.00	.9712	.6656	50.23
2	2	.01	.90	10.00	.8906	.7220	29.53
3	.5	.01	.90	10.00	.8578	.7810	28.04
4	.5	.01	.90	0.00	.8240	.8471	18.05
5	1	.01	.90	0.00	.8090	.8471	18.05
6	.5	.01	.90	10.00	.8039	.7810	28.04
7	.5	.01	.90	10.00	.7750	.7220	29.53
8	2	.01	.90	20.00	.7116	.6656	50.23

$A = .7664$ $\tau = 19.50$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.01	.90	10.00	.9673	.7012	42.60
2	.5	.01	.90	10.00	.9325	.7489	33.53
3	.5	.01	.90	10.00	.8961	.7670	30.38
4	2	.01	.90	10.00	.8376	.7822	27.85
5	1	.01	.90	10.00	.7976	.7822	27.85
6	2	.01	.90	10.00	.7461	.7670	30.38
7	.5	.01	.90	10.00	.7182	.7489	33.53
8	.5	.01	.90	10.00	.6729	.7012	42.60

$A = .7498$ $\tau = 20.99$

k	w_k	λ_k	σ_k	r_k	z_k	A_k	$E[T_k]$
1	—	.005	.95	0.00	.9925	.8504	35.19
2	1	.005	.95	10.00	.9836	.8319	40.40
3	1	.005	.95	20.00	.9678	.8144	45.57
4	1	.005	.95	40.00	.9466	.7722	59.01
5	1	.005	.95	40.00	.9165	.7722	59.01
6	1	.005	.95	20.00	.8879	.8144	45.57
7	1	.005	.95	10.00	.8690	.8319	40.40
8	1	.005	.95	0.00	.8493	.8504	35.19

$A = .8172$ $\tau = 16.83$

6. Numerical results

The provision of a comprehensive set of tables for the performance measures discussed would make prohibitive demands on space, since each station has its own value for λ_i, r_i, σ_i, and w_i. It is of course possible to write a general computer program in which these values are input as data for each of the N stations. Provided that $N \leqslant 9$, such a program will give results in a reasonable time on commonly available computer systems. The numerical results, given in Table 1, are hence restricted to a few particular examples, although there is no serious problem in extending these. They are further restricted to the symmetric case which makes the calculations easier, although the model is not so realistic.

As has been indicated, even in the case of homogeneous stations, the important performance measures are not homogeneous. On the basis that average availability is of importance, the general indication is that it is preferable to put the least reliable machines, with high repair times, near the middle of the line, although this point perhaps requires more detailed investigation over a wider range of values of the parameters.

The more difficult problem is to extend the value of N beyond 9 (say). Apart from the special case mentioned in Section 5, the method requires the solution of 2^{N+1} equations for the probabilities $\pi(\vec{u})$ and $\pi(\vec{v})$. The frustrating element is that all the performance measures can be calculated from a knowledge of the $2N$ probabilities $z_1, \ldots, z_N$, and $z'_1, \ldots, z'_N$. However it appears that the only way to calculate these is to use the $\pi(\vec{u})$ and $\pi(\vec{v})$ as in (3.2)–(3.3).

In a practical context, the model has been used to predict the overall availability of machines set out in rows on a factory floor. The lack of homogeneity arose in this situation from a mix of old and new machines which had different breakdown rates and which, because of the differing nature of the tasks they were performing, also had different repair times. Most of the breakdowns in this situation arose from a single failure mode, and the assumption of random breakdowns appeared to be reasonable. Repair was generally a routine procedure, and again constant repair time seemed to be a reasonable assumption. If the repair time is variable, then the mathematical modelling becomes intractable at the time of writing.

7. Acknowledgements

Dr. J. Sztrik was partially supported by the Hungarian National Science Foundation under grant OTKA/648/91. The authors are also grateful for a suggestion from John Bentin which has resulted in a much improved presentation of this work.

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